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# On the scaling behavior of the cosmological constant and the possible existence of new forces and new light degrees of freedom

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## ABSTRACT

A large value of the cosmological constant (CC) is induced in the Standard Model (SM) of Elementary Particle Physics because of Spontaneous Symmetry Breaking. To provide a small value of the observable CC one has to introduce the vacuum term which cancels the induced one at some point in the very far infrared cosmic scale. Starting from this point we investigate whether the cancellation is preserved at different energy scales. We find that the running of the Higgs mass, couplings and the vacuum term inevitably result in a scaling dependence of the observable value. As a consequence one meets a nonzero CC at an energy scale comparable to the typical electron neutrino mass suggested by some experiments, and the order of magnitude of this constant is roughly the one derived from recent supernovae observations. However the sign of it is negative – opposite to what is suggested by these observations. This discrepancy may be a hint of the existence of an extra very light scalar, perhaps a Cosmon-like dilaton, which should essentially decouple from the SM Lagrangian, but that it nevertheless could mediate new macroscopic forces in the submillimeter range.

# 1 Introduction

According to the modern understanding, Particle Physics can be successfully described by the Standard Model (SM) of the strong and electroweak interactions, which could perhaps be approximately valid till some Grand Unification scale  $M_X \sim 10^{16} GeV$  or even up to the Planck scale  $M_P \simeq 10^{19} GeV$ . The Spontaneous Symmetry Breaking (SSB) through the Higgs mechanism is a crucial ingredient of the SM as it makes the weak gauge vector fields massive while preserving the gauge invariance. As a by-product, however, the introduction of a scalar Higgs potential and the SSB leads to a non-vanishing value of the vacuum energy which can be interpreted as a direct contribution to the cosmological constant (CC) in the Einstein-Hilbert action for the gravitational field [1]:

$$S_{gr} = - \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G} R + \Lambda \right). \quad (1)$$

Indeed, the SM contains a doublet of complex scalar fields  $\Phi$ . In the ground (vacuum) state the expectation value (VEV) of  $\Phi^\dagger \Phi$  will be denoted  $\langle \Phi^\dagger \Phi \rangle \equiv \frac{1}{2} \phi^2$ , where  $\phi$  is a classical scalar field. The corresponding classical potential reads

$$V_{cl} = -\frac{1}{2} m^2 \phi^2 + \frac{\lambda}{8} \phi^4. \quad (2)$$

Shifting the original field  $\phi \rightarrow H + v$  such that the physical scalar field  $H$  has zero VEV one obtains the physical mass of the Higgs boson:  $M_H = \sqrt{2} m$ . Minimization of the potential (2) yields the SSB relation:

$$\phi = \sqrt{\frac{2m^2}{\lambda}} \equiv v \quad \text{and} \quad \lambda = \frac{M_H^2}{v^2}. \quad (3)$$

The natural (experimental) electroweak mass scale in the SM is  $M_F \equiv G_F^{-1/2} \simeq 293 GeV$ , where  $G_F = 1.166 \cdot 10^{-5} GeV^{-2}$  is the physically measurable Fermi constant from muon decay. The VEV of  $\phi$  can be written entirely in terms of it:  $v = 2^{-1/4} M_F \simeq 246 GeV$ . Similarly, from eq. (3) one obtains the scalar coupling constant completely determined at that scale:  $\lambda = \sqrt{2} G_F M_H^2$ . All in all, even lacking at present of direct experimental evidence of the Higgs boson, the indirect effects from precision observables show that the SM is already a very much successful Quantum Field Theory at the Fermi scale [2]. However, from (2) and (3) one obtains the following induced value for the CC, at the tree-level, that goes over to eq.(1):

$$\Lambda_{ind} = \langle V_{cl} \rangle = -\frac{m^4}{2\lambda}. \quad (4)$$

If we apply the current numerical bound  $M_H \gtrsim 100 GeV$  from LEP II, then the value of  $|\Lambda_{ind}|$  is 55 orders of magnitude larger than the observed upper bound for the CC – typically this bound is  $\Lambda \lesssim 10^{-47} GeV^4$  [1]. This is the Cosmological Constant Problem in the SM of electroweak interactions.

To cure the CC problem one usually introduces a vacuum cosmological term  $\Lambda_v$  with opposite sign. The physical (observable) CC that finally goes into eq.(1) is then

$$\Lambda_{ph} = \Lambda_{ind} + \Lambda_v, \quad (5)$$

and indeed the aforementioned astronomical upper bound applies only to this quantity.

Some observations are in order. Since the consideration concerns gravity, it would be natural to start from the renormalizable theory in curved space-time [3]. Such a theory contains the  $\xi R\varphi^2$  term, and this term seems to affect the SSB relations. This doesn't happen, however, unless we imply a huge value of the parameter  $\xi$ . In order to see this, we remind that the value of the curvature is defined (at least for small energies, where higher order terms in the gravity action may be safely omitted) from the Einstein equations, and therefore  $R \sim 8\pi G\Lambda_{ph}$  or  $R \sim 8\pi GT_\mu^\mu$ . As  $G = 1/M_P^2 \approx 10^{-38} GeV^{-2}$  and the observable density of the matter in the Universe and current observational limits on the cosmological constant are both of the same order, the actual value of the curvature scalar is very small ( $R \sim 10^{-84} GeV^2$ ) – a reflex that a very small CC is equivalent to the almost flatness of present day space-time – and so the SSB relations do not need to be altered. In fact, the relevant dimensionless parameter that measures the deviation is  $\xi R/m^2 < \xi 10^{-88} \ll 1$  unless  $\xi$  is extremely large. Similar considerations apply at the Fermi epoch where the picture remains unaltered unless  $\xi \gg 10^{33}$ .

The introduction of the vacuum cosmological term is dictated also by the requirement of renormalizability of the massive theory. Thus, the CC problem is neither the existence of this term nor its renormalization, but the need of the extremely precise choice of the corresponding normalization condition  $\Lambda_{ph} = 0$  in the very far infrared (IR). We assume that this point lies somewhere after our epoch. At present, nobody knows the reason why the two terms on the right-hand side of (5) – plus perhaps some non-perturbative QCD contribution – should cancel each other with such an unnaturally big accuracy at that point.

## 2 The running of the cosmological constant

There were a number of attempts to solve the CC problem although no one of them was successful enough [1]. In this letter we do not try to understand the whys and wherefores of the original fine-tuning at very low-energies <sup>1</sup>, but rather the physical consequences of its quantum instability. We address this problem by evaluating the quantum effects, namely the ones that can be computed from the Renormalization Group (RG) method in the framework of the effective field theory. It will be shown that the leading contributions to the running of the physical CC (5) cancel and that the sub-leading contributions produce a negative CC at the energies above the mass of the lightest fermion particle. This situation, however, can be very different at higher energy domains near the Fermi scale  $M_F$  or beyond [5]. In what follows, we are going to discuss the renormalization group equations (RGE's) for quantities such as masses, couplings and CC. For the investigation of the running of  $\Lambda$  (the same concerns the running of the gravitational constant  $G$ , which will be studied elsewhere [5]) one can always work with the beta-functions without taking into account the classical dimensions for these quantities. The reason is that these

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<sup>1</sup>Models have been suggested of finite theories in curved space-time where  $\Lambda$  naturally vanishes in the far IR. See e.g. [4] and references therein. Their relation with the SM, though, is far from clear.

appear as powers of the dimensionless ratio  $(\mu/\mu_0)$ . However the running constants acquire physical sense only when they are substituted into Einstein equations. Since these equations are homogeneous in dimensions the previous ratio cancels automatically in them.

As we have already seen, the physical CC is nothing but the sum of two parts: induced and vacuum, each of them satisfying its own RGE. Our initial framework here will be just the SM, but later on we shall introduce some extra stuff. The one-loop RGE for the vacuum part gains contributions from all massive fields, and can be computed in a straightforward way by explicit evaluation of the vacuum bubble [3]. In particular, the contribution from the complex Higgs doublet  $\Phi$  and the fermions is (for  $\mu > M_F$ )

$$(4\pi)^2 \frac{d\Lambda_v}{dt} = 2m^4 - 2 \sum_i N_i m_i^4, \quad \Lambda_v(0) = \Lambda_0, \quad (6)$$

where the sum is taken over all the fermions with masses  $m_i$ . Here  $t = \ln \mu$ , and  $N_i = 1, 3$  for leptons and quarks respectively.  $\Lambda_v$  has to be normalized at the very far IR cosmic scale, in order to provide the precise cancellation of the induced contribution in (5).

We shall study (6) and subsequent RGE's in some approximation. This includes, in particular, omitting all higher loop contributions, and involves some application of the effective field theory approach. Indeed, this approximation works better at lower energies. On the other hand, at these energies the running of masses and couplings is weak, and therefore this running can be disregarded together with the higher loops contributions. Thus, to estimate the running of  $\Lambda_v$ , we assume that  $m$  and  $m_i$  take fixed values characteristic of the Fermi scale.

On the other hand, the RGE for the induced CC follows, according to eq. (4), from the general RGE's for the scalar mass  $m$  and the coupling  $\lambda$  [6]. In the SM, the latter read:

$$\begin{aligned} (4\pi)^2 \frac{dm^2}{dt} &= m^2 \left( 6\lambda - \frac{9}{2} g^2 - \frac{3}{2} g'^2 + 2 \sum_{i=q,l} N_i h_i^2 \right), & m^2(0) &= m_F^2, \\ (4\pi)^2 \frac{d\lambda}{dt} &= 12\lambda^2 - 9\lambda g^2 - 3\lambda g'^2 + \frac{9}{4} g^4 + \frac{3}{2} g^2 g'^2 + \frac{3}{4} g'^4 \\ &\quad + 4 \sum_{i=q,l} N_i h_i^2 (\lambda - h_i^2) & \lambda(0) &= \lambda_F. \end{aligned} \quad (7)$$

Here  $q = (u, d, \dots, t)$  and  $l = (e, \mu, \tau, \nu_e, \nu_\tau, \nu_\mu)$  label the type of spinor (quark and lepton) fields of the SM,  $g, g'$  are the  $SU(2)_L$  and  $U(1)_Y$  gauge couplings, and  $h_{l,q}$  are Yukawa coupling constants for the fermion fields. The boundary conditions for the renormalization group flow are imposed at the Fermi scale  $M_F$  for all the parameters, with the important exception of  $\Lambda_v$ . Then,  $g_F^2 \approx 0.4$  and  $g_F'^2 \approx 0.12$ . In considering the equation for  $\Lambda_{ind}$  we shall follow the same strategy as described above for eq. (6) – that is, we shall disregard the second order effects related to the running of other parameters and attribute to all couplings and masses their constant values at the Fermi scale.

A crucial point concerning the RGE's is the energy scale where they actually apply. This is especially important in dealing with the Cosmological Constant Problem, since this problem is not seen at the Fermi epoch, but at the present epoch, i.e. at energies far below the Standard Model scale  $M_F$ . Now, the beta-functions  $\beta_{\Lambda_v}, \beta_m, \beta_{h_i}, \beta_\lambda \dots$

governing those RGE's depend on the number of active degrees of freedom. These are the number of fields whose associated particles have a mass below the energy scale  $\mu$  that we are considering, because at sufficiently small energies one can invoke the decoupling of the heavier degrees of freedom [7]. In this work we are interested in the scaling behavior of the physical  $\Lambda_{ph}$  at very low energies. The importance of this energy domain is due to a recent analysis of a set of high-redshift supernovae of Type Ia [8], which suggest that  $\Lambda_{ph} > 0$  at the 99% *C.L.* and it also pinpoints a value for  $\Lambda_{ph}$  of the order of a few times the matter density  $\rho_m$ . Specifically, from the combined use of these data and CMBR measurements [9] the following ranges are singled out:  $\Omega_m = 0.24 \pm 0.10$  and  $\Omega_\Lambda = 0.62 \pm 0.16$ , where  $\Omega_m = \rho_m/\rho_c$  and  $\Omega_\Lambda = \Lambda_{ph}/\rho_c$ , with  $\rho_c = 3H_0^2/8\pi G \simeq 8.1 h_0^2 \times 10^{-47} GeV^4$  the critical density. Taking  $h_0 = 0.65 \pm 0.10$  [10] this gives, roughly,  $\Lambda_{ph} \simeq (2 \pm 1) \times 10^{-47} GeV^4$ .

### 3 Cosmological constant and neutrino masses

In view of the previous consideration one can put forward the possibility that this positive CC may be an effect of quantum scaling at an energy scale  $\mu$  above the very far IR where  $\Lambda_{ph}$  is zero. Following this line of thought, we may expect that the lightest degrees of freedom of the SM, namely the neutrinos, are the only ones involved to determine  $\Lambda_{ph}$  at nearby IR points where we perform our measurements. In fact, since the standard solution of the CC problem supposes an extremely exact fine tuning in (5), and since both vacuum and induced terms satisfy their own RGE's, it shouldn't be a great surprise if their exact cancellation breaks down by the running. However, one may fear that this breaking can have a disastrous effect. In the sense that, being both parts of (5) many orders of magnitude greater than their sum, any violation of the fine tuning might produce a huge CC at any neighboring IR scale, a fact which would be blatantly inconsistent with cosmology. Indeed this could make all the approach based on a fine-tuning untenable.

Fortunately, nothing like that occurs. For the SM provides an automatic cancellation mechanism between the running of  $\Lambda_{ind}$  and  $\Lambda_v$ , so that at very small energy scales only the second order effects remain. These effects really drive  $\Lambda_{ph}$  away from zero (if we suppose that it started at zero value), but the order of magnitude of this modification is small in the IR and can be consistent with the supernovae observations. However, the sign of the resulting  $\Lambda_{ph}$  is opposite to the one derived from these observations, and one has to look for additional physical input to cure this drawback.

To see all this in our RG approach, let us consider the energy region well below the electron mass, where the only relevant particles are neutrinos  $\nu_i$ . Then the effects from the other massive fields – Higgs boson, quarks, leptons and massive vectors – in equations (6) and (7) can be dropped. There are no contributions from photons and gluons, so that we arrive at the following simplified equation for the vacuum term:

$$(4\pi)^2 \frac{d\Lambda_v}{dt} = - \sum_j 2m_j^4, \quad j = \nu_e, \nu_\mu, \nu_\tau, \quad \Lambda_v(0) = \Lambda_0. \quad (8)$$

One can, indeed, rewrite the last equation in terms of the Yukawa couplings  $h_j$ . We assume that neutrinos get masses through the usual mechanism in the SM, and therefore  $m_j = h_j v/\sqrt{2}$ . Of course, the quantities  $h_j$  are very tiny, many orders smaller than any other coupling of the SM.

Next we explicitly check that the RGE for the induced counterpart  $\Lambda_{ind}$  exhibits (in the SM) an important cancellation, namely that of the leading  $m^4 h_j^2 / \lambda$  terms, and one is left with  $m^4 h_j^4 / \lambda^2$  which are much smaller for neutrinos. Indeed, from (4) and (7)

$$\frac{d\Lambda_{ind}}{dt} = \frac{m^4}{2\lambda^2} \frac{d\lambda}{dt} - \frac{m^2}{\lambda} \frac{dm^2}{dt} = -\frac{1}{(4\pi)^2} \cdot \frac{2m^4}{\lambda^2} \cdot \sum_j h_j^4 = -\frac{2}{(4\pi)^2} \sum_j m_j^4, \quad (9)$$

where we have used the fact that in the SM the coefficient  $m^4 h_j^4 / \lambda^2$  is nothing but  $m_j^4$ . If the fermion masses  $m_j$  are very small, the quantum scaling evolution of the induced CC is slow enough not to disturb the standard cosmological scenario. Moreover, at very low energies, we see that the RGE's for the vacuum and induced CC are identical and turn out to be very simple. Therefore the running of the physical CC satisfies, in this energy domain, the equation

$$(4\pi)^2 \frac{d\Lambda_{ph}}{dt} = -4 \sum_j m_j^4, \quad \Lambda_{ph}(0) = \Lambda_{ph}(IR). \quad (10)$$

Here  $\Lambda_{ph}(IR)$  is the value of the physical CC in the very far infrared, which we assumed  $\Lambda_{ph}(IR) = 0$ . Therefore, according to our framework, a non-vanishing value of  $\Lambda_{ph}$  is to be generated from the lightest degrees of freedom available in the Universe.

In the minimal SM, only neutrinos could have masses small enough such that their contribution to  $|\Lambda_{ph}|$  lies in the correct ballpark. However, the spectrum of masses and mixing angles predicted by different neutrino experiments do entail some restrictions on the parameters [11]. In particular, an additional (sterile) neutrino  $\nu_s$  is frequently invoked in many texture models for the neutrino masses. (In the strict SM case,  $\nu_s$  would simply be a RH neutrino.) There are a variety of possibilities, but one can easily check that in general in these models not all neutrino species (if any) could be adequate to generate a value for  $|\Lambda_{ph}|$  in the desired range. Nonetheless, as it was shown in [12], there is one scenario that could perhaps resolve both the Solar Neutrino Problem and the Atmospheric Neutrino Problem. In this model both the electron neutrino and the sterile neutrino are much lighter than the rest:  $m_{\nu_e}, m_{\nu_s} \ll m_{\nu_\mu}, m_{\nu_\tau}$ . The ANP solution would follow from  $\nu_\mu - \nu_\tau$  oscillations, and the SNP from taking the masses  $m_{\nu_e}, m_{\nu_s}$  in the ballpark suggested by the nonadiabatic MSW mechanism (resonant oscillation inside the solar medium) [11]: namely, a region  $\delta m_{ex}^2 = (0.3 - 1) \times 10^{-5} GeV^2$  at small mixing angle, and a region of  $\delta m_{ex}^2$  roughly between  $(1.5 \times 10^{-5} - 1 \times 10^{-3}) GeV^2$  at nearly maximum mixing angle.

In this framework, not only the SNP and ANP problems could be solved, but it could also afford the necessary hot dark matter component, provided  $m_{\nu_\mu}, m_{\nu_\tau}$  are almost degenerate and in the  $eV$  range. This in turn could make the LSND result not that “untenable” with respect to the remaining neutrino experiments [11].

Remarkably, such an scenario is roughly compatible with, say,  $m_{\nu_e} = (2 - 4) \times 10^{-3} eV$  and so, by integrating (10), one easily finds  $|\Lambda_{ph}| \lesssim 10^{-47} GeV^4$  i.e. a number below, but very near, the right range. As a matter of fact, in the region of maximal mixing one can amply achieve the desired  $|\Lambda_{ph}|$ . Notice that in this picture the other two neutrino species are  $10^2 - 10^3$  times heavier, and hence would not enter the RG analysis of  $\Lambda_{ph}$  in the far IR. But of course they would enter at earlier cosmological times – and at some point also the remaining SM particles, which determine the evolution of the CC at the Fermi epoch [5]. Now, in spite of the fact that this neutrino scenario is compatible with the correct order of magnitude of  $\Lambda_{ph}$ , the sign of this parameter as predicted by eq. (10) is negative, contrary to the famous supernovae result [8].

## 4 New light scalar fields and new forces

Intriguingly enough, the onerous sign from neutrinos could be remedied by introducing an extra (real) light scalar field  $S$  of non-vanishing mass  $m_S$ . We may also compute its contribution to the vacuum bubble. Then the RGE for the physical cosmological constant becomes

$$(4\pi)^2 \frac{d\Lambda_{ph}}{dt} = \beta_\Lambda \equiv \frac{1}{2} m_S^4 - 4 \sum_i m_{\nu_i}^4; \quad \Lambda_{ph}(0) = 0, \quad (11)$$

where the sum extends over the electron neutrino and an sterile neutrino according to the favorite model mentioned above. For definiteness, we may assume that their average mass is  $m_\nu \sim 2 \cdot 10^{-3} eV$ .

Let us briefly discuss possible ways to realize this program. Of course one could invoke the existence of a “just-so” scalar field<sup>2</sup> with a mass of order  $m_S \gtrsim 2 m_\nu$  in order to flip the sign of the overall  $\beta_\Lambda$ –function in (11). In actual fact,  $m_S$  must be a few times larger than  $m_\nu$  ( $m_S \gtrsim 4 m_\nu$ ) in order to insure that after integrating eq.(11) a value for  $\Lambda_{ph}$  obtains in the right ballpark. As for the interactions, this scalar should effectively be an “sterile scalar” – very weakly interacting. We recall that there are stringent phenomenological constraints on light scalars that severely restrain their couplings and ranges [14]. In particular, a light Higgs boson is completely ruled out both by low-energy and LEP experiments [15].

On the other hand, an axion-like pseudoscalar is not precisely what we want here. For, although axion mass windows presently constrain it to be in a mass range whose upper bound is  $\sim 10^{-3} eV$  [14]– and so still of the order of the ultralight neutrinos mentioned above –, the problem with axions in this context is that an essential relation like eq. (9) is very peculiar to the Higgs structure of the SM. Therefore there is no guarantee that it could be preserved – without fine-tuning of the parameters – in the framework of general two-Higgs-doublet models. Similarly, we understand that additional Higgs bosons are also used in some specific models in the literature in order to implement the correct pattern of neutrino masses mentioned above through radiative corrections [12]. However, we do not commit ourselves to the underlying structure of these models. In our framework the Higgs sector at the Fermi scale should effectively behave as that of the SM.

Therefore, if we choose not to depart much from the RG structure of the SM, we have to resort to alternative scenarios. For example, we could entertain the possibility that the necessary light scalar  $S$  is a dilaton-like field. Such a field can be identified to be the Goldstone boson that emerges from a non-linearly realized formulation of global dilatation-invariance in the SM. Namely, one assumes that the dilatation symmetry of the SM Lagrangian is spontaneously broken at some high energy scale  $M$ . Then any operator in the SM Lagrangian (including the vacuum term) is made invariant by multiplying it by a suitable power of  $e^{S/M}$ . The scale variance of that operator is compensated by a corresponding shift  $S \rightarrow S + cM_X$  characteristic of a non-linearly realized Goldstone mode. It is understood that a kinetic term for the non-linear field  $S$  and the Einstein piece (1) also enter the total Lagrangian, and that the gravitational part is made dilatation invariant following the same philosophy, which in this case implies that the Ricci scalar

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<sup>2</sup>The existence of an *ad hoc* scalar particle could be proposed only to saturate the bound on  $\Lambda$  [13], independently of neutrinos –if they are assumed massless. However, all hints seem to point towards light massive neutrinos and  $\Lambda > 0$ . Therefore, in a consistent picture these must enter the game.

$R$  is replaced by  $R e^{2S/M}$ . Notwithstanding, the scale symmetry of the SM plus Einstein Lagrangians so constructed is explicitly broken by quantum effects. These give rise to the trace anomaly, so that the Goldstone boson  $S$  is not exactly massless. The properties of this boson are very similar to the ones first described in Ref. [16] and for this reason we still dub it a “Cosmon”, because it may help us to understand the present status of the CC problem<sup>3</sup>. It should, however, be clear that we adopt here a rather different point of view than in the previous references. After all,  $\Lambda_{ph}$  was measured to be nonzero! Still, the present field  $S$  has similarly qualitative, though quantitatively different, phenomenological consequences, as it will be shown below.

The Cosmon mass,  $m_S$ , is determined by the interplay between the hard scale defined by the trace anomaly and the scale  $M$  of SSB of dilatation symmetry. The anomalous trace of the energy-momentum tensor is given by the sum

$$\Theta_\mu^\mu = \frac{\beta(g_s)}{2g_s} F_a^\mu F_\mu^a + \Theta_W + \Theta_G, \quad (12)$$

where the first term is the QCD part,  $\Theta_W$  is the weak component and  $\Theta_G$  a possible gravitational contribution. The last two terms should be completely negligible at very low energies in our effective field theory approach. Following [16], the Cosmon mass ensues from the equation of motion of  $S$ , which is determined by the anomalous Ward identity generated by the covariant derivative of the dilatation current:

$$\nabla_\mu J^\mu = \Theta_\mu^\mu. \quad (13)$$

For instance, the scalar contribution to  $J^\mu$  reads

$$J^\mu = \sqrt{-g} \{ (1 + 12h) M e^{2S/M} \partial^\mu S + \Phi \partial^\mu \Phi \}. \quad (14)$$

Notice that the particular piece in this dilatation current, namely the one that is proportional to

$$h \equiv \frac{1}{16\pi} \frac{M_P^2}{M^2}, \quad (15)$$

is a direct contribution from the dilatation-invariant scalar curvature term. The linearized equation (13) will become the field equation of motion for  $S$  if there is a stable value  $S = S_0$  where  $\langle \Theta_\mu^\mu \rangle$  vanishes. The  $S$ -dependence of the latter could appear e.g. through a change of gauge  $\beta$ -functions at a higher scale [16]. For  $S_0/M \ll 1$ , the structure of the SM will remain basically unaltered up to terms of  $\mathcal{O}(1/M)$  which should be small if  $M$  is sufficiently large. After expanding around  $S = S_0$ , and neglecting the electroweak and gravitational contributions, simple dimensional analysis tells us that the mass (squared) of  $S$  is essentially given by

$$m_S^2 = \frac{\Lambda_{QCD}^4}{M^2}, \quad (16)$$

where  $\Lambda_{QCD} = \mathcal{O}(100) \text{ MeV}$  is the intrinsic QCD scale. Taking  $M \gtrsim 10^{10} \text{ GeV}$  eq. (16) gives  $m_S \lesssim \mathcal{O}(10^{-3} - 10^{-2}) \text{ eV}$  for the Cosmon mass<sup>4</sup>. Thus we see that, within the

<sup>3</sup>For other developments and applications of the Cosmon model, see [17, 18].

<sup>4</sup>One could also obtain this same value through e.g.  $m_S^2 = M_F^4/M$  by choosing  $M$  near the more popular GUT scale  $M_X = 10^{16} \text{ GeV}$ . However, it is difficult to imagine how the electroweak dilatation anomaly could dominate in our approach in the far IR. So we do not expect the scale  $M_F$  to be involved. Moreover, at variance with the existence of an intrinsic, RG-invariant, scale  $\Lambda_{QCD}$  in QCD, there is no obvious counterpart in the electroweak theory—unless one postulates it!. All in all it is quite natural that the non-perturbative QCD effects become relevant at low energy.



Cosmon context, the correct order of magnitude for the mass of the necessary scalar can be achieved if one assumes that dilatation symmetry is spontaneously broken at some intermediate GUT scale:  $M_F \ll M \ll M_X$ . The order of magnitude of  $M$  is pinned down by the positivity condition  $\beta_\Lambda > 0$  in eq.(11), and so ultimately by the value of the lightest neutrino masses and of the cosmological constant.

Interestingly enough, due to the anomaly, the linearized effective Lagrangian contains a residual piece of gravitational strength. After canonical normalization of the Cosmon kinetic term – following from (13) and (14) – this piece reads

$$\mathcal{L}_S = \frac{S}{M\sqrt{1+12h}} \Theta_\mu^\mu = f G^{1/2} S \Theta_\mu^\mu, \quad (17)$$

where

$$f = \sqrt{\frac{16\pi h}{1+12h}}. \quad (18)$$

We emphasize that an important virtue of the Cosmon-dilaton picture is that this scalar “completely” decouples from the SM Lagrangian, as can be easily demonstrated by a conformal transformation of all the matter fields,  $\varphi \rightarrow e^{-DS/M} \varphi$  (according to their canonical dimension  $D$ ) and a Weyl rescaling of the metric tensor:  $g_{\mu\nu} \rightarrow e^{2S/M} g_{\mu\nu}$ . In this conformal basis one may check explicitly that the only couplings of  $S$  are derivative couplings – as in fact could be expected from a (pseudo-) Goldstone boson – with the exception of the anomalous term (17), which in this basis comes out rescaled by a factor of  $e^{-4S/M}$ .

Consider next the (gravitational-like) macroscopic force mediated by Cosmon exchange. For a nucleon  $N$ , the “Cosmon charge” is given by  $f$  times the nucleonic matrix element of the operator (12). We do not expect that within a nucleon there is any significant remnant of the electroweak and gravitational anomaly terms. Therefore,

$$Q = f \langle N | \Theta_\mu^\mu | N \rangle = f \left\langle N \left| \frac{\beta(g_s)}{2g_s} F_a^\mu F_\mu^a + m_q \gamma(g_s) \bar{q} q \right| N \right\rangle, \quad (19)$$

In the presence of a nucleus we have added in (19) the anomalous dimension contribution from the quark mass operator. Thus, formally, we recover exactly the same expression for the macroscopic force carried by the old Cosmon model [16]. Except that in the present instance, the Compton wavelength of this Cosmon is much shorter:  $\lambda_S = 1/m_S \gtrsim (.2 - .02)$  mm. Therefore we expect it to mediate a submillimeter range macroscopic force  $\sim \alpha/r^2$  supersimposed on the normal gravitational interaction. Furthermore, because it is the anomalous trace of the energy-momentum tensor – rather than the trace of the full energy-momentum tensor – that enters eq.(19), this submillimeter new interaction is not only of pure gravitational mass-dependent nature, but it can also carry (hierarchically weaker) composition-dependent (baryon number and isospin number dependent) components – similarly as in [16]. The relative strength of the dominant component (the mass-dependent one) of the new interaction with respect to gravity is given by  $\alpha = f^2/4\pi$ . Of course its value hinges on the value of the parameter  $f$  in eq.(19), which in turn depends on the ratio between the scales  $M$  and  $M_P$ . If the former is, as we already suggested above, an intermediate GUT scale several orders of magnitude below  $M_X$ , then from eqs.(15) and (18) we get  $\alpha = 1/3$  – to a very good approximation – and the new submillimetric interaction is comparable to gravity!.

We remark that, in contradistinction to the approach of [16], here we do not propose any relation between the VEV's of the full energy-momentum tensor and its anomalous part, and so no corresponding dynamical adjustment mechanism has been called upon that could lead to previously described inconsistencies [1]. The Cosmon field  $S$  serves here only to make the case of a light scalar degree of freedom that could realize the positivity condition on the RHS of eq.(11) while at the same time to evade all known present bounds on light scalars – and still leave a physical imprint of its existence!. And all this can in principle be achieved without perturbing the effective structure of the SM Lagrangian.

## 5 Discussion and conclusions

From the previous considerations we may outline the following heuristic picture. The reason why there are very light (massive) degrees of freedom in our Universe could be related to the (measured) existence of a very small residual cosmological constant  $\Lambda_{ph}$  in the late epochs of its history. We think of this constant as a running parameter in a Quantum Field Theory of matter and gravitation. Although this theory has not been clearly uncovered, yet, we have explored the consistency of the  $\Lambda_{ph} = 0$  initial condition by applying the RG method to the SM, coupled to Einstein gravity, as the effective low-energy theory. We have shown that the RG structure of the SM is such that the light degrees of freedom do not lead to a “runaway value” of  $\Lambda_{ph}$  in neighboring infrared points. In fact, the presently measured value of  $\Lambda_{ph}$  could be explained in terms of these light particles. But of course there are other difficulties. Inherent to this approach is the issue of imposing the original strong boundary condition on the RGE of  $\Lambda_{ph}$  at a certain point in the very far cosmological infrared. We cannot justify this matter on first principles, but we take it as a natural fact based on the observed smallness of the measured CC. Notice, as a daring remark, that the parameters involved in the cancellation, being those defined at the very far IR, are in a sense unobservable. However, what we possibly observe nowadays is the value of  $\Lambda_{ph}(t)$  – given by eq.(5) – at an earlier IR point in cosmological time, that is, before  $\Lambda_{ph}(t)$  “red-shifts” until its complete extinction, perhaps attracted by some very far IR-stable fixed point – corresponding to  $\beta_\Lambda > 0$  in eq.(11). If we would further entertain the possibility that the strong boundary condition at issue just defines that IR-stable fixed point, then this could explain the natural flow of  $\Lambda_{ph}(t)$  towards  $\Lambda_{ph}(0) = 0$ .

In our approach, far from addressing the details of the fundamental underlying theory, we have studied the phenomenological consequences associated to a radiative departure from the initial IR condition. To this end, we have adopted the effective field theoretical assumption that the various contributions at higher and higher energies depend on the progressive excitation of heavier and heavier degrees of freedom, of masses  $M_\alpha$ , when  $\mu > M_\alpha$ . The ultimate justification for applying this procedure to  $\Lambda_{ph}$  and using only the lightest degrees of freedom is the reasonable order of the predictions for  $\Lambda_{ind}$  and finally for the physical  $\Lambda_{ph}$  at the energies far below the electron mass. We stress that the deviation from this effective method would produce a large value of the CC incompatible with the Standard Cosmological Model (Cf. [5]). So at this stage of knowledge, and in view of the nice interconnection that it could lead between the recent cosmological observations and the last experimental findings in neutrino physics, we rather think of it as a theoretical

Ansatz that could be useful inasmuch as it leads to new, testable, physical consequences. For example, recently there has been much interest in short-range macroscopic forces generated by string-like models and in general by models that introduce extra, compact, dimensions at the millimeter scale [19]. In the last reference it is emphasized that these short-range forces are not in contradiction with existing macroscopic experiments [14]. From our line of thought we have found an additional, though completely different, motivation to look for new gravitational forces in Nature: namely, they are associated to the existence of new light degrees of freedom reflecting a non-zero value of the cosmological constant.

Part of this light spectrum is familiar, like the lightest SM neutrino (possibly the electron neutrino), and maybe also an accompanying sterile neutrino of similar mass into which the latter can oscillate. We have emphasized that this scenario could be consistent with all neutrino experiments. However, the measured sign  $\Lambda_{ph} > 0$  could be an indication that a light scalar  $S$  of a mass similar to that of these neutrinos (in fact a few times larger) should exist in order to tilt the balance of the predicted cosmological constant into the positive range, and at the same time to secure the sign  $\beta_\Lambda > 0$  that guarantees IR stability of the RG flow. If the necessary scalar is sterile (as it is the case for the extra neutrino), then there would be no feasible experimental method to detect it, and thus no chance to substantiate this approach. Indeed, one can explore various models for the light scalar  $S$  with the necessary magnitude of the mass. For instance, it can be some string-induced dilaton (see e.g. [20]) or the anomaly-induced scalar (see e.g. [21]), both with a mass generated by some dynamical mechanism. In this respect we recall that all versions of string gravity boil down, in the low-energy limit, to Einstein gravity plus a new ingredient: the dilaton. Therefore, on very general grounds, a dilaton type of scalar  $S$  is a quite a generic object in Quantum Field Theory below  $M_P$ , and under appropriate circumstances it could effectively behave as a Cosmon of the type described above. If this would be the case, or if an alternative theoretical framework would lead to a non-linear Goldstone-mode realization of dilatation symmetry below some intermediate unification scale, the structure of the SM would remain essentially unaltered, but there would appear a new macroscopic force in Nature, whose ultimate reason could stem from the observed non-vanishing value of the cosmological constant. The potential detection, in future experiments [14], of new gravitational-like forces down to the submillimeter range – perhaps being attractive and exhibiting a composition-dependent nature – could be the sought-after “smoking gun” hinting at the possibility of this whole scenario.

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